

CONSERVATION, CHOICE, AND THE CONCURRENT FIXED-RATIO SCHEDULE

NEIL SHAPIRO AND JAMES ALLISON¹

INDIANA UNIVERSITY

Five rats got all of their water in daily 60-minute sessions. Two levers and a water spout were freely available throughout baseline sessions. Contingency sessions offered a choice between two alternative fixed-ratio components, in the form of a choice between the two levers. Each component required a specified number of lever presses for access to the spout, and then a specified number of licks for another choice between components. Given the observed relative frequency, the absolute frequency of selecting each component was predicted accurately by assuming that the subject conserved between baseline and contingency the total amount of a dimension attributable to lever pressing and licking. Several quantitative models for predicting relative frequency were examined. The best of these assumed that the subject would show a nonexclusive preference for the component requiring fewer lever presses.

Key words: conservation, choice, concurrent fixed-ratio, quantitative models, lever press, drinking, rats

The two conservation models already published (Allison, 1976) are for schedules that happen to place narrower limits on the subject's behavior than do many other schedules in common use. One model is applicable to the simple schedule composed of a single fixed-ratio component—requiring, for example, some fixed number of lever presses for a fixed number of licks at a water spout. The other model is applicable to the mixed schedule composed of two such components arranged in regular alternation, such as a schedule requiring four lever presses for 60 licks (Component 1), and then eight lever presses for 120 licks (Component 2), followed by another performance of Component 1, another of Component 2, and so on. Here, we introduce a third model, designed specifically for the concurrent fixed-ratio schedule. By allowing the subject to perform either or both of two alternative fixed-ratio components, the concurrent schedule offers an element of choice not present in the other two schedules, and which therefore has no representation in their models. To explain

the structure of the new model, it is convenient to begin with a brief review of the other two.

Conservation models are expressed in terms of the responses—usually of two kinds—that are specified by the contingency schedule, and two types of session having the same duration. In the paired-operant baseline session (Timberlake and Allison, 1974), the two responses can occur freely throughout. For example, if the two responses were lever pressing and drinking, both the lever and the water spout would be presented at the outset of the baseline session, and would remain freely available throughout; both would retract as soon as the fixed session time had elapsed. The other type is the reciprocal contingency session (Allison, 1971; Timberlake and Allison, 1974), in which the subject must perform each response for the opportunity to perform the other. An example would be a session begun by introducing the lever: after a fixed number of lever presses, the lever retracts and the water spout appears; after a fixed number of licks at the water spout the spout retracts, and the lever reappears. Lever or spout retracts as soon as the fixed session time is up.²

¹The research was supported in part by a grant to the first author from the Honors Division, Indiana University. We thank Sarah Bottjer for help in collecting the data, and Eliot Hearst and Mark Wozny for comments on drafts of the manuscript. Reprints may be obtained from James Allison, Department of Psychology, Indiana University, Bloomington, Indiana 47401.

²See Timberlake and Allison (1974) for a discussion comparing the reciprocal contingency with the more conventional procedure in which lever pressing is required for access to the water spout, but drinking is

All three of the models discussed here assert the same theoretical premise: the total amount of some dimension attributable to the two responses is conserved by the subject as between the baseline session and the contingency session. Symbolizing one of the two responses as Response *i* and the other as Response *c*, consider a simple fixed-ratio schedule that specifies *I* units of Response *i*, and *C* units of Response *c*. For example, suppose the schedule requires *I* lever presses for access to the water spout, and *C* licks at the spout for renewed access to the lever. The conservation model for this simple fixed-ratio schedule can be expressed as

$$N(kI + C) = kO_i + O_c \quad (1),$$

where the dependent variable *N* refers to the number of times in the contingency session the subject completes the requirement sequence *I* + *C*, and *O_i* and *O_c* refer to the total amounts of the two responses performed in the paired-operant baseline session. The dimensional parameter *k* is a unit-free ratio, the amount of the dimension entailed in performing one unit of Response *i* relative to the amount entailed in performing one unit of Response *c*.

Some hypothetical cases will clarify the interpretation of Equation 1. For example, suppose that the dimension conserved is energy expenditure—which may indeed be true of at least some pairs of responses, such as feeding activity and nonfeeding activity (Morrison, 1968). Suppose further that Responses *i* and *c*, lever pressing and licking the spout, are measured in terms of the number of lever presses and the number of licks. If one lever press happens to expend four times as much energy as one lick at the water spout, then *k* = 4 in Equation 1.

The right-hand side of the equation, *kO_i* + *O_c*, would give the total amount of energy expended in pressing the lever and licking the spout in the baseline condition, expressed in units of Response *c*. The latter statement

would be true even if the two responses were measured in different units, such as number of lever presses and time spent licking the spout. The parameter *k* would then refer to the amount of energy expended by one lever press relative to the amount expended by 1 sec of licking. The numerical value of *k* would then be less than four, as 1 sec of licking normally involves about five licks, and would therefore entail more energy expenditure than one lick.

The left-hand side of Equation 1 refers to the contingency session. Recall that *N* is the number of times the subject completes the sequence *I* + *C*. Then, the total number of lever presses in the contingency session is the product *NI*, plus any more presses that may have occurred if the session happened to end amidst a partial completion of the requirement *I*. Similarly, the total number of licks is *NC*, plus any more licks done in partial completion of *C*. It follows that the left-hand side of Equation 1, *kNI* + *NC*, would approximate the total amount of energy expended in pressing the lever and licking the spout in the contingency condition, expressed in units of Response *c*—in the present example, lick-joules.

Useful predictions can be derived from Equation 1 whatever the units of measurement may be. Each of the two responses can be measured in terms of any handy unit, provided the unit is used consistently in baseline and contingency, so that *O_i* and *I* refer to the same unit, and *O_c* and *C* refer to the same unit. By solving Equation 1 for the dependent variable *N*, then multiplying both sides by *I*, the approximate total amount of Response *i* in the contingency session, *NI*, is expressed as a function of the schedule parameters *I* and *C*, the baseline parameters *O_i* and *O_c*, and the dimensional parameter *k*. A similar equation for Response *c* is obtained by solving for *N*, and multiplying both sides by *C*.

A recent review of experimental literature on simple fixed-ratio schedules indicates considerable empirical support for several theoretical implications of Equation 1 (Allison, 1976). The same article reports evidence in support of the model intended for a mixed schedule composed of two components arranged in regular alternation. Letting the subscripts 1 and 2 designate Components 1 and 2, the model for the mixed schedule is

not required for access to the lever because the lever remains available throughout the session. See Allison (1976) for a discussion of the widespread procedure in which the schedule is defined in terms of a specified amount of lever pressing and the delivery of a specified amount of water, as opposed to a specified amount of drinking.

$$N(KI_1 + C_1 + kI_2 + C_2) = kO_i + O_c \quad (2).$$

The present investigation tested a conservation model for concurrent fixed-ratio schedules. Here, the rat was allowed to choose between two alternative fixed-ratio components at the outset of the session, and after each completion of the chosen sequence of requirements—either I_1 lever presses followed by C_1 licks at the water spout, or I_2 presses followed by C_2 licks. The contingency session was begun by introducing two levers, left and right. If Component 1 happened to be scheduled on the left lever and Component 2 on the right for that particular session, the first press on the left lever caused the right to retract. The first press on the left lever thereby functioned as a selection response, and also counted as the first response in the requirement I_1 . In like manner, the first response on the right lever caused the left to retract, thereby selecting Component 2, and counted as the first response in the requirement I_2 . Upon each completion of C_1 or C_2 licks, the water spout retracted and both levers reappeared for another choice between the two components.

Unlike the two models discussed earlier, each of which involved only one dependent variable N , the model for this concurrent schedule must involve two. The dependent variable N_1 signifies the number of times the subject selects and performs the sequence of requirements specified by Component 1, namely $I_1 + C_1$. The other dependent variable, N_2 , signifies the number of times the subject selects and performs the sequence specified by Component 2, namely $I_2 + C_2$. Note that N_1 and N_2 are experimentally independent of each other. Then, the model for this concurrent fixed-ratio schedule can be expressed as

$$N_1(kI_1 + C_1) + N_2(kI_2 + C_2) = kO_i + O_c \quad (3).$$

Equation 3 asserts, as do Equations 1 and 2, that the subject conserves between the baseline condition modelled on the right-hand side of the equation, and the contingency condition modelled on the left, the total amount of the dimension apportioned to Responses i and c .

Equation 3 can be solved for either dependent variable in terms of the other dependent variable, the dimensional parameter k , the schedule parameters, and the baseline parameters. Because the schedule parameters are set

by the experimenter and the baseline parameters are measured experimentally, the model could be used to predict either dependent variable, given the numerical value of k and the numerical value of the other dependent variable.

There is an alternative solution to this problem of prediction that does not require the observed numerical value of N_1 or N_2 . This more adroit solution makes use of the observed selection ratio, N_1/N_2 . Letting y signify the observed selection ratio, $y = N_1/N_2$, from which

$$N_1 = N_2y \quad (4).$$

Substituting N_2y for N_1 in Equation 3 and solving for N_2 , the model predicts that

$$N_2 = \frac{kO_i + O_c}{y(kI_1 + C_1) + kI_2 + C_2} \quad (5).$$

It should be clear that this use of the observed selection ratio would in no way bias our test of the conservation model. Any particular ratio—say $N_1/N_2 = 2$ —would be consistent with many values of N_1 and N_2 , but only one combination would also be consistent with Equation 3—say 20 and 10, but not 8 and 4, 26 and 13, or any other combination in which N_1 is twice N_2 .

The one remaining problem in using the model to predict N_1 and N_2 is that of assigning a numerical value to the dimensional parameter k . One solution involves testing the individual with several different concurrent fixed-ratio schedules. In conjunction with the model and the baseline data, the subject's performance on each schedule provides the basis for one estimate of k . Solving Equation 3 for k ,

$$k = \frac{O_c - (N_1C_1 + N_2C_2)}{(N_1I_1 + N_2I_2) - O_i} \quad (6).$$

The numerator in Equation 6 refers in the present context to the total number of licks obtained in baseline, minus the total number of licks obtained with a particular concurrent fixed-ratio schedule. The denominator refers to the total number of lever presses obtained with that schedule, minus the total number of lever presses obtained in baseline. Equation 6 can be used to estimate k for each schedule, and the mean of the various estimates can then be used to predict N_1 and N_2 for each of the individual's schedules. If the subject does not behave as specified by Equation 3, then the

subject's mean value of k will not provide accurate predictions of N_1 and N_2 across the different schedules.

Another solution involves an estimate of k which is completely independent of the data that the model is meant to predict. One way of obtaining such an estimate is to depend on data from an earlier experiment, which used the same apparatus as the present experiment but a different sample of rats and a different kind of schedule (Allison, 1976).

Both of these methods of estimating k were tried here. In addition, the variety of schedules used permitted comparative tests of several alternative models for predicting selection ratios. These models of selection were used in conjunction with the conservation model in an attempt to predict N_1 and N_2 independently of the observed selection ratios.

METHOD

Subjects

Five male Wistar rats purchased from Harlan Industries (Cumberland, Indiana) served. Four were experimentally naive, and began the experiment when approximately 100 days old. The fifth, 150 days old, had been used to test the apparatus.

Apparatus

The experimental chamber was an aluminum box 25 cm square and 19 cm high. Two retractable aluminum levers mounted on the rear wall extended 1.5 cm into the chamber. When retracted, their frontal surfaces, 3.5 by 1.5 cm, were flush with the wall. The levers were 8 cm apart, center to center, and 8 cm above the chamber floor. Lever presses were registered by microswitch closures; each lever had a force requirement of 0.177 N.

A 1.5-cm hole midway between the levers and 5.5 cm above the chamber floor permitted access to a metal water spout positioned 1 cm outside the rear chamber wall. Access to the spout was controlled by means of a metal shutter interposed between the outside surface of the wall and the spout; closing this shutter covered the access hole. Licks on the spout were monitored by means of a drinkometer circuit (BRS/LVE, DO-201), grounded to the stainless-steel floor of the chamber.

Three electric motors controlled by solid-state logic modules moved the two levers and

the shutter independently of one another. The total number of licks and the total number of lever presses performed in each component were recorded on electromechanical counters in an adjacent room. Two other counters recorded the number of times the subject selected each component of the concurrent fixed-ratio schedule. The chamber was illuminated by three green 1-W lamps, one above each lever and one above the spout.

Procedure

One week before the experiment, daily access to water in the home cage was restricted to the 60-min period when experimental sessions would subsequently take place. During the experiment, all drinking was confined to the experimental chamber. Food was always available in both the home cage and the experimental chamber, in the form of Purina laboratory chow pellets.

Each subject received a daily 60-min experimental session, beginning with a precontingency baseline phase. This baseline phase was followed by a contingency phase, which used one of the schedules specified in Table 1, and the contingency phase was followed in turn by a postcontingency baseline phase. Each additional schedule called for an additional contingency phase followed by another postcontingency baseline phase. The subject received the various schedules in the order listed in Table 1.

Levers and spout were presented at the beginning of each baseline session by extending both levers and opening the shutter simultaneously. Levers and spout remained freely available throughout the baseline session, at the end of which the levers were retracted and the shutter was closed simultaneously. Precontingency baseline sessions were conducted until the individual subject stabilized. The stability criterion was a sliding block of four sessions, in which there was no monotonic increase or decrease in the total amount of either response, and no individual session in which the total number of licks deviated from the block mean by more than 10% of the block mean. Subjects met this stability criterion in 11 to 21 sessions.

Each contingency phase consisted of two parts, pretraining and testing. The phase began with four pretraining sessions with a mixed schedule, in which the two fixed-ratio

components were arranged in regular alternation. The object of pretraining was to familiarize the subject with the two components before offering a choice between them, and to reduce any lever-position bias. Components were counterbalanced with respect to position over the four-session pretraining block: Component 1 appeared on the left in Sessions 1 and 4, and on the right in Sessions 2 and 3. Each pretraining session was begun by extending the left lever. In Sessions 1 and 4, when Component 1 was on the left, the left lever retracted and the shutter opened immediately after a cumulative total of I_1 presses. Immediately after a cumulative total of C_1 licks, the shutter closed and the right lever extended. The right lever retracted and the shutter opened immediately after I_2 presses. After C_2 licks, the shutter closed and the left lever extended for another performance of the sequence $I_1 + C_1 + I_2 + C_2$. In Sessions 2 and 3, when Component 2 was on the left, the required sequence was $I_2 + C_2 + I_1 + C_1$.

The same fixed-ratio components, now arranged as a concurrent schedule, were used in the test sessions that followed the four pretraining sessions. Both levers were extended at the beginning of each test session. The first press on either lever constituted a selection response, which caused the other lever to retract immediately. Upon a cumulative total of I_1 or I_2 presses—including the selection response—the lever retracted, and the shutter opened to allow access to the spout. Upon a cumulative total of C_1 or C_2 licks, the shutter closed and both levers extended for another choice between components.

Components were counterbalanced in testing as they were in pretraining: Component 1 appeared on the left in Sessions 1 and 4, on the right in Sessions 2 and 3. These four-session blocks were repeated until the individual subject met two stability criteria based on the mean of the first two sessions and the mean of the last two sessions in the block. The first criterion applied to the total frequency of selection, $N_1 + N_2$: the four means from two consecutive blocks could show no monotonic increase or decrease, and none of the four could deviate from the grand mean of the four by more than 10%. The second criterion applied to the number of times Component 1 was selected relative to all selections, $N_1/(N_1 + N_2)$: the four means from two consecutive

blocks could show no monotonic increase or decrease, and none of the four means could differ from their grand mean by more than 0.1. As soon as the subject met these two stability criteria, four postcontingency baseline sessions were conducted. The subject was then eligible for another contingency phase with another schedule.

One purpose of the experiment was to test several models for predicting the selection ratio N_1/N_2 . The schedules, which are shown in Table 1, incorporated several features necessary for a comprehensive comparative test. One notable feature is that the two components that constituted three of the schedules required different numbers of presses and licks, but identical press:lick ratios (2:30 *versus* 4:60, 2:30 *versus* 8:120, and 4:60 *versus* 8:120). The two components that constituted the four remaining schedules required different press:lick ratios. Two of these four required a different number of lever presses but the same number of licks (2:60 *versus* 4:60, and 2:60 *versus* 8:60), whereas the other two required the same number of presses but a different number of licks (2:30 *versus* 2:60, and 2:30 *versus* 2:120).

RESULTS AND DISCUSSION

Except as noted, all analyses were based on the data shown in Table 1, which are means calculated from four sessions: the last four sessions of precontingency baseline, the four baseline sessions conducted after each test, and the last four sessions of each test. Estimates of k shown in Table 1 were calculated by substituting into Equation 6 the precontingency baseline values and the test values shown in the table. For example, in the first row of the table, $k = (5348 - 2066 - 3158)/(129 + 114 - 4) = 0.519$. The values of N_1 and N_2 shown in the table were based on readings of the two selection counters. They can only be approximated by dividing the nominal schedule requirement, I_1 or I_2 , into the obtained number of lever presses shown in the table, because each rat sometimes managed to do one press more than the nominal requirement before the lever was fully retracted.

The predicted value of N_2 , the number of times the individual subject selected and performed Component 2 of any particular schedule, was calculated by substituting into Equa-

Table 1
Lever Presses and Licks Observed in Baseline (*O*₁ and *O*₂), Scheduled and Observed in Test Sessions^a

Rat	Precontingency		Postcontingency		Number of Test Sessions	Scheduled		Observed		k	N ₁	N ₂	N ₁ /(N ₁ + N ₂)
	O ₁	O _c	O ₁	O _c		I ₁ :C ₁	I ₂ :C ₂	Component 1	Component 2				
R-1	4	5348	17	5242	28	2:30	2:60	129:2066	114:3158	0.519	62	53	0.483
	3	531	24	635				26 473	19 488		12	8	0.064
			1	5236	12	2:30	2:120	112:1633	68:3648	0.267	55	31	0.506
			1	299				4 71	9 646		2	6	0.036
R-2			12	5603	16	2:60	4:60	105:2956	157:2248	0.558	49	38	0.561
			21	382				3 157	24 354		3	6	0.039
	9	4822	32	4626	24	2:30	4:60	175:2186	187:2496	0.397	72	41	0.541
	5	222	18	240				21 155	17 178		5	3	0.055
R-3			20	4797	34	2:30	8:120	128:1813	202:2933	0.233	60	24	0.513
			16	796				23 276	17 229		9	2	0.035
			36	4418	8	4:60	8:120	162:2219	173:2465	0.423	37	21	0.524
			67	338				7 93	12 135		2	1	0.035
R-4	5	6230	6	6056	24	2:60	4:60	135:3960	146:2060	0.761	66	34	0.661
	4	218	8	467				24 803	30 515		13	9	0.105
			17	5901	30	2:60	8:60	139:4287	211:1468	1.377	71	24	0.738
			18	462				18 501	44 319		8	5	0.068
R-5			6	6276	12	2:30	2:60	112:1598	150:4429	0.790	52	73	0.382
			9	312				22 363	5 153		12	2	0.079
	7	5867	37	5802	26	2:30	4:60	191:2390	186:2513	2.605	80	43	0.546
	1	390	50	519				17 241	11 139		8	3	0.020
R-6			31	5257	16	2:30	8:120	153:1900	218:3120	2.327	62	26	0.559
			48	877				6 134	54 782		5	6	0.068
			18	5569	8	4:60	8:120	156:2081	213:2948	2.315	35	25	0.536
			25	509				18 248	8 62		4	1	0.052
R-7			8	5744	8	2:60	8:60	134:3523	212:1480	2.549	59	25	0.702
			8	255				23 489	35 274		8	5	0.062
	4	5223	17	5481	16	2:30	2:120	107:1526	67:3676	0.124	50	31	0.491
	4	266	21	538				6 153	5 247		5	2	0.027

^aItalicized entries are standard deviations that reflect variability across the four sessions independently of position.

tion 5 the nominal requirements of the schedule, the individual's mean k as calculated across schedules, the individual's precontingency baselines, and the individual's observed selection ratio for that particular schedule. (All of the quantities used in calculating the predicted value of N_2 can be obtained from Table 1.) The predicted value of N_1 was calculated by substituting into Equation 4 the observed selection ratio, and the predicted value of N_2 . Because Rat R-5 was tested with only one schedule, all of R-5's data would have been exhausted in estimating k from Equation 6; R-5 was therefore omitted from this particular analysis.

Values obtained for N_1 and N_2 agreed closely with values predicted by the conservation model. The product-moment correlation was extremely high in the case of both N_1 ($r = 0.998$) and N_2 ($r = 0.999$). Goodness-of-fit tests revealed no significant difference between predicted and obtained values of N_1 , $\chi^2 (13) = 0.35$, $p > 0.99$, or N_2 , $\chi^2 (13) = 0.20$,

$p > 0.99$. Values predicted and obtained are compared in Panel A of Figure 1, which also shows the individual values of k used in calculating the predictions. The continuous diagonal lines in Figure 1 are predicted functions, and the broken lines are least-squares regression lines.

A second solution to the problem of assigning a numerical value to k was to use a single estimate of k for all rats and all schedules: an estimate based on a different sample of rats, tested in the same apparatus but with a different type of schedule and a different unit of measurement—time spent responding—rather than the number of presses and the number of licks. Although conservation theory does not imply that k will necessarily be invariant across individuals or units of measurement, this second solution holds the attraction of an empirical test of such invariances. Another of its attractions is that this second solution would use the conservation model in a way that is unmistakably predictive, because it

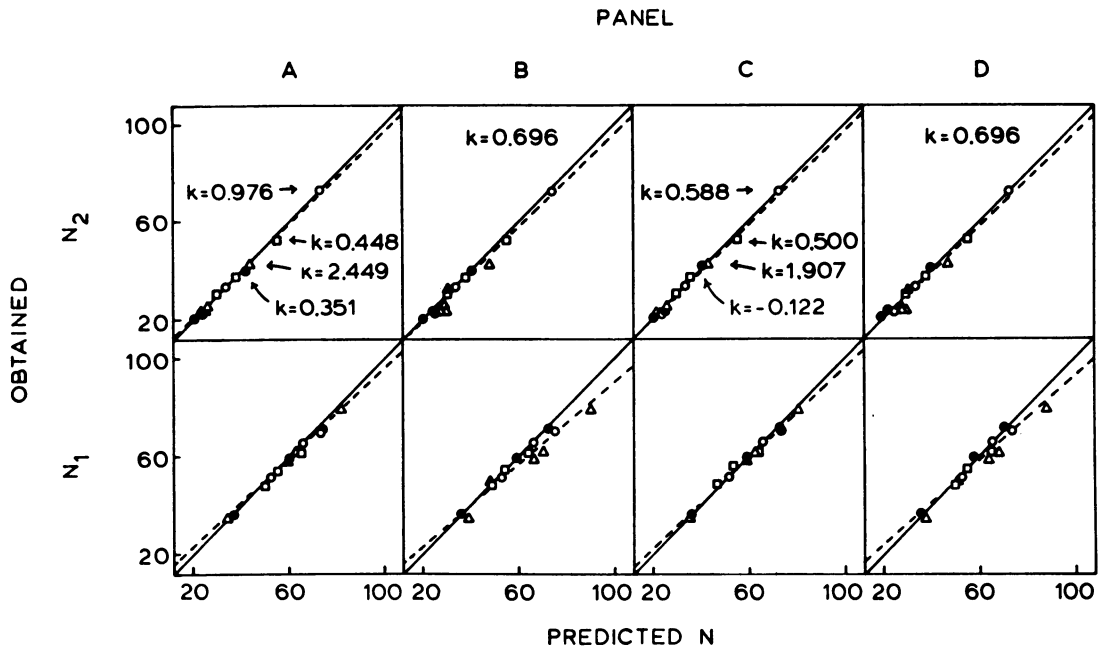


Fig. 1. Number of times each component was selected (N_1 and N_2), compared with values predicted by the conservation model on the basis of observed selection ratios for individuals R-1 (open square), R-2 (closed circle), R-3 (open circle), R-4 (open triangle), and R-5 (closed triangle, Panels B and D). Continuous lines are predicted functions; broken lines are least-squares regression lines. Predictions shown in Panels A and B were based on the individual's precontingency baselines. Predictions shown in Panels C and D were based on the mean of the individual's pre- and postcontingency baselines. Predictions shown in Panels A and C were based on the individual's mean value of k as calculated across schedules. Those shown in Panels B and D were based on a single value for all individuals and all schedules, $k = 0.696$, as estimated from another experiment (see text for further explanation).

would use none of the present data for the estimate of k . A related advantage over the first solution is that Rat R-5, although tested with only one schedule, could then be included in the analysis: R-5's data would not be exhausted in estimating k , because no part of its data would be used in estimating k .

The second solution was made possible by the earlier experiment on mixed schedules, which provided an estimate of $k = 0.696$ (Allison, 1976, p. 192). The predictions based on this single value of k for all subjects and all schedules appear in Panel B of Figure 1, which reveals close agreement with values obtained. In the case of N_1 the correlation was $r = 0.97$, and the difference between predicted and obtained values was not significant statistically, $\chi^2(14) = 3.56$, $p > 0.99$. Analysis of N_2 also revealed a high correlation, $r = 0.99$, and no significant difference between predicted and obtained values, $\chi^2(14) = 1.72$, $p > 0.99$. Further analyses explored the extent to which k could be varied before the model would yield predictions unmistakably at odds with the data. The threshold value of k at which all 14 predicted values of N_1 and all 14 of N_2 were lower than the values obtained was about 3.1. The threshold at which all predicted values were higher than obtained was about -0.7 .

The predictions shown in Panels A and B of Figure 1 were based on the precontingency baselines. Also computed were corresponding predictions based on the mean of each individual's pre- and postcontingency baselines, although the postcontingency data were questionable because they did not necessarily represent stable values. These additional predictions were not expected to differ appreciably from the corresponding predictions in Panels A and B, because there was no systematic difference between the k 's based on the precontingency baselines and those based on the mean of the pre- and postcontingency baselines. The group means as calculated across the five rats were $k = 0.870$ (precontingency), and $k = 0.759$ (pre- and postcontingency baselines combined). The k based on the precontingency data was the larger member of the pair of k 's for three rats, and the smaller member for the other two rats (R-1 and R-5). The predictions based on the mean of the pre- and postcontingency baselines are shown in Panels C and D of Figure 1. Visual comparison of corresponding panels suggests that Panel C

would support essentially the same conclusions as Panel A, and D the same as B. Correlational analyses and goodness-of-fit tests fully confirmed this impression. Further analyses related to Panel D showed that the threshold value of k at which all predicted values of N_1 and N_2 were lower than obtained was approximately 2.6. The threshold at which all predicted values were higher than obtained was about -0.7 .

In summary, the behavior conformed closely to Equation 3, the conservation model that would apply to performance on the concurrent fixed-ratio schedules used here. We arrived at this conclusion by showing that the conservation model made accurate predictions of the two dependent variables, N_1 and N_2 , given the observed selection ratio, N_1/N_2 . It follows that if another model could predict the selection ratio accurately, the two models in conjunction could make accurate predictions of N_1 and N_2 .

Several variables were considered as possible predictors of the dependent variable, $y = N_1/N_2$. One of these predictor variables was the amount of response deprivation (Timberlake and Allison, 1974) represented by Component 1, relative to the amount of response deprivation represented by Component 2. Each of the components used in the present experiment deprived the subject of the licking response, in the sense that the total number of licks in the contingency session would have fallen short of the number performed in the baseline session, had the subject performed in contingency only the baseline number of lever presses. In symbolic terms, it was true of every component that NC would have been less than O_c , had NI been equal to O_i . However, some components represented a greater degree of response deprivation than others. For any particular individual, a component that allowed a relatively small number of licks per lever press represented a relatively large amount of response deprivation. The degree to which the component deprives the subject of Response c has been denoted D_c , where

$$D_c = O_c - (CO_i/I)$$

(Timberlake and Allison, 1974). Assuming that both components deprive the subject of Response c , the amount of deprivation represented by Component 1 relative to the amount

represented by Component 2 can be denoted D_{c1}/D_{c2} , where

$$\frac{D_{c1}}{D_{c2}} = \frac{O_c - (C_1 O_i / I_1)}{O_c - (C_2 O_i / I_2)} = \frac{I_2 (I_1 O_c - C_1 O_i)}{I_1 (I_2 O_c - C_2 O_i)} \quad (7).$$

Equation 7 defines one of the alternatives considered as a predictor variable, x , in the function $y = f(x)$.

Another alternative was inspired by the concept of psychological distance to reward (Duncan and Fantino, 1972). In deciding which of the two responses should be viewed as the reward, we were guided by recent developments (Allison and Timberlake, 1973; Allison and Timberlake, 1974; Timberlake and Allison, 1974), which suggest that reward might best be defined in terms of response deprivation. If reward is defined as contingent access to Response c , given that the schedule deprives the subject of Response c , then the psychological distance to reward is an increasing function of the instrumental requirement I . Assuming that both components deprive the subject of Response c , the relative distance to reward, denoted $\text{Distance}_{c1}/\text{Distance}_{c2}$, is

$$\frac{\text{Distance}_{c1}}{\text{Distance}_{c2}} = \frac{I_1}{I_2} \quad (8).$$

A third alternative was based on the concept of response cost. In the present context, the ratio I/C expresses the response cost of Response c in terms of the number of lever presses required per lick (*cf.* Schwartz, 1969). Relative cost of Response c is denoted $\text{Cost}_{c1}/\text{Cost}_{c2}$, where

$$\frac{\text{Cost}_{c1}}{\text{Cost}_{c2}} = \frac{I_1/C_1}{I_2/C_2} = \frac{I_1 C_2}{I_2 C_1} \quad (9).$$

A fourth variable was defined in terms of the number of times the conserving subject would have to perform the sequence $I + C$ if the component were presented alone as a simple fixed-ratio schedule, $N = (kO_i + O_c)/(kI + C)$. The relative number of performances is denoted N_1/N_2 , where

$$\frac{N_1}{N_2} = \frac{(kO_i + O_c)/(kI_1 + C_1)}{(kO_i + O_c)/(kI_2 + C_2)} = \frac{kI_2 + C_2}{kI_1 + C_1} \quad (10).$$

Because we had no compelling preconceptions about the mathematical form of the equation relating the dependent variable y and the independent variable x , our regres-

sion analysis tried four conventional functions with each independent variable: linear, exponential, logarithmic, and power. Precontingency baselines were used for independent variables defined in terms of O_i and O_c (Equations 7 and 10), and a uniform $k = 0.696$ was used for Equation 10.

The results of the regression analysis, presented in Table 2, revealed that the logarithmic function was generally superior to the other three functions, and that the simplest variable, relative distance to reward, was invariably superior to the other three predictor variables. The selection ratio, N_1/N_2 , was inversely related to the relative distance to reward, I_1/I_2 . The regression equation, $N_1/N_2 = 1.23 - 0.92 \ln(I_1/I_2)$, accounted for about 61% of the variance.

In a series of analyses parallel to those shown in Panel B of Figure 1 (precontingency baselines, $k = 0.696$), values of N_1/N_2 were predicted from the logarithmic relative distance equation and substituted into Equations 4 and 5. The predicted values of N_1 and N_2 that resulted from this procedure were fairly close to the values obtained. The correlation between values predicted and obtained was $r = 0.80$ in the case of N_1 , and $r = 0.89$ in the case of N_2 . There was no significant difference between values predicted and obtained, $\chi^2(14) = 21.37$, $p > 0.05$ (N_1), and $\chi^2(14) = 17.36$, $p > 0.20$ (N_2).

Further analyses revealed significant differences between obtained values of N_1 or N_2 and the values predicted by using the other three independent variables as predictors of N_1/N_2 in the logarithmic form of the regression equations. The relative response deprivation predictor defined by Equation 7 resulted in predictions that differed significantly from both N_1 , $\chi^2(14) = 32.93$, $p < 0.01$, and N_2 , $\chi^2(14) = 24.19$, $p < 0.05$. The same was true of the relative N predictor defined by Equation 10, $\chi^2(14) = 33.19$, $p < 0.01$ in the case of N_1 , and $\chi^2(14) = 24.19$, $p < 0.05$ in the case of N_2 . The relative cost predictor defined by Equation 9 gave a similar outcome: the difference between values predicted and obtained was highly reliable in the case of N_1 , $\chi^2(14) = 30.88$, $p < 0.01$, and approached the conventional level of significance in the case of N_2 , $\chi^2(14) = 23.05$, $p = 0.06$.

The distance model was clearly superior to the others, being the only one of the four se-

Table 2
Regression Analyses with $y = N_1/N_2$, and Various Definitions of x

Function	Constants and Correlational Statistics	Definition of x			
		Deprivation ^a	Distance ^b	Cost ^c	N^d
Linear $y = a + bx$	a	16.76	2.70	2.08	1.68
	b	-14.91	-1.49	-0.18	0.06
	r^2	0.20	0.53	0.14	0.01
	r	-0.45	-0.73	-0.37	0.12
Exponential $y = a_e^{bx}$	a	5719.76	2.92	1.98	1.54
	b	-8.09	-0.90	-0.10	0.05
	r^2	0.15	0.50	0.11	0.02
	r	-0.39	-0.71	-0.33	0.15
Logarithmic $y = a + b \ln(x)$	a	1.85	1.23	1.84	1.80
	b	-15.13	-0.92	-0.40	0.02
	r^2	0.20	0.61	0.29	0.00
	r	-0.45	-0.78	-0.53	0.00
Power $y = ax^b$	a	1.74	1.22	1.74	1.67
	b	-8.22	-0.53	-0.22	0.04
	r^2	0.16	0.54	0.23	0.00
	r	-0.40	-0.74	-0.48	0.00

^aSee Equation 7.
^bSee Equation 8.
^cSee Equation 9.
^dSee Equation 10.

lection models whose predictions, made in conjunction with the conservation model, encountered no significant failure. Did the distance model leave any room for improvement? This question was answered in the affirmative by a comparison of correlations between values predicted and obtained for N_1 and N_2 (Ferguson, 1959). The correlations based on the observed values of N_1/N_2 were significantly higher than the correlations based on values of N_1/N_2 predicted by means of the distance model. In the case of N_2 , both correlations were quite high, but even there the correlation based on the observed value of N_1/N_2 was significantly higher than the correlation based on the value predicted from the distance model, $r = 0.99$, *versus* $r = 0.95$, $t(11) = 3.33$, $p < 0.01$. The difference was also significant in the case of N_1 , $r = 0.97$ *versus* $r = 0.80$, $t(11) = 4.13$, $p < 0.01$.

The fact that I_1/I_2 was more closely related to N_1/N_2 than any predictor variable that incorporated C_1 and C_2 should not be taken to mean that the subjects were indifferent to the number of licks afforded by the two alternative components. The reason is that the mean of N_1 , relative to the mean of N_2 , can be misleading as an index of preference between components if the conserving subject happens to have a strong preference for one of the two

levers. For example, consider a hypothetical rat that conserves in accordance with Equation 3, is perfectly indifferent as between the two components, and has an overwhelming position bias in favor of the left-hand lever. Under the stated hypothetical conditions, the rat will direct all of its lever pressing to the same lever in all test sessions. If each component appears on the left in half of the sessions, as in the present experiment, then in half of the sessions N_1 will be zero, and N_2 will be equal to $(kO_i + O_c)/(kI_2 + C_2)$. In the other half of the sessions, N_2 will be zero, and N_1 will be equal to $(kO_i + O_c)/(kI_1 + C_1)$. As calculated across sessions, mean N_1 /mean N_2 will then be $(kI_2 + C_2)/(kI_1 + C_1)$. It follows that mean N_1 /mean N_2 may or may not be 1, even if the rat is perfectly indifferent as between the two components. In the conserving, component-indifferent rat with an overwhelming positional bias, mean N_1 /mean N_2 will be 1 only if $(kI_2 + C_2)$ and $(kI_1 + C_1)$ are equal. This complication does not arise in interpreting another measure, mean $N_1/(N_1 + N_2)$. Under the stated hypothetical conditions, mean $N_1/(N_1 + N_2)$ will be 0.5 independently of the relation between $(kI_2 + C_2)$ and $(kI_1 + C_1)$, because this measure is independent of the number of times the subject selects and performs the two components.

The last column on the right in Table 1 presents the preference measure, $N_1/(N_1 + N_2)$. This measure shows that preference was influenced by differences in C as well as differences in I , but that I carried more weight than C . The table shows that the smaller value of I was selected in all of the four cases in which the two values of C were equal. The larger value of C was selected in three of the four cases in which the two values of I were equal. When the two I/C ratios were equal, but the I 's and the C 's differed, the smaller value of I was selected in all six cases, despite the fact that this smaller value of I was packaged with the smaller value of C .³

GENERAL DISCUSSION

Our major new finding was that conservation theory gave a good account of performance on a schedule that offers an element of choice not offered by the kinds of schedule already modelled successfully (Allison, 1976). The conservation model expressed as Equation 3 provided a close fit to performance on our concurrent fixed-ratio schedules. Given the ob-

served relative frequency of selection, the model predicted accurately the absolute number of times the rat would select each of the two components. An important secondary finding was that the conservation model also did quite well without benefit of the observed relative frequencies, if they were replaced by values predicted by various models for selection. Of the four selection models studied here, the best one took the form of an inverse logarithmic relation between the relative frequency of selection and the relative number of lever presses required for the contingent opportunity to lick water.

There is a fifth model of selection whose predictions would be based on the relative number of lever presses required by the two components, but its predictions would disagree with values we observed when the two components differed in the amount of lever pressing required. In discussing his adaptation of Herrnstein's matching model to concurrent ratio schedules, Pear (1975) showed that his formulation would predict exclusive responding on one component if the two components differed in the number of lever presses required. Contrary to this prediction, our data showed that each component was selected quite frequently (see Table 1, Rows 5-16 and 19-26). This liberal sampling of both components is clearly evident in both measures, $N_1/(N_1 + N_2)$ as well as N_1/N_2 . The closest approach to exclusive responding was a three-to-one preference for the component that required fewer presses when the other component required four times as many.

Informal observations made during the test sessions suggest that this strong departure from exclusive preference is probably not attributable to our pretraining procedure. Subjects did not show the pattern of alternation between components, which was the hallmark of the behavior in pretraining, except sometimes for a short period in the first test session after pretraining—a session that never figured in the data shown in Table 1. A small part of the departure from exclusive preference might be attributable to the precautions we took to avoid confusing a lever-position bias with a preference for the component. Because we varied the position of the component across sessions in counterbalanced order, it was perhaps more likely that the first response in the session would be an inadvertent selection of

³The adequacy of Equation 3 as an account of the present data did not depend on our having used the observed value of N_1/N_2 , rather than the observed value of $N_1/(N_1 + N_2)$. In another series of analyses parallel to those shown in Figure 1, we defined y' as

$$y' = N_1/(N_1 + N_2),$$

from which

$$N_1 = (N_1 + N_2)y' \quad (11),$$

and

$$N_2 = (N_1 + N_2)(1 - y') \quad (12).$$

If the expressions on the right-hand sides of Equations 11 and 12 are substituted for N_1 and N_2 in Equation 3, we have

$$y'(N_1 + N_2)(kI_1 + C_1) + (1 - y')(N_1 + N_2)(kI_2 + C_2) = kO_1 + O_c \quad (13).$$

Solving Equation 13 for the dependent variable, $N_1 + N_2$,

$$N_1 + N_2 = \frac{kO_1 + O_c}{y'(kI_1 + C_1) + (1 - y')(kI_2 + C_2)} \quad (14).$$

Values predicted from Equation 14 were compared with values obtained for $N_1 + N_2$ by means of regression analyses and goodness-of-fit tests. The analysis showed that the model's prediction of $N_1 + N_2$ on the basis of $y' = N_1/(N_1 + N_2)$ was nearly as accurate as the model's prediction of N_1 and N_2 on the basis of $y = N_1/N_2$. The correlations between values predicted and obtained ranged from $r = 0.93$ to $r = 0.95$, and all χ^2 tests were nonsignificant.

the less-preferred component than if we had not taken the trouble to unconfound the component and its position. Nonetheless, Table 1 would still reveal much sampling of each component, even if N_1 and N_2 were each reduced by considerably more than the one response that would logically suffice to identify both the location and the requirements of each component. For example, suppose that instead of needing to sample each lever only once, the rat needed considerably more experience—say, 20 selections of each lever—to learn the location and the requirements of each component at the outset of any particular session. We could take this slow learning into account by reducing each session's observed value of N_1 and N_2 by 20 each. Upon trying this massive "correction" on the rat and the schedule that had originally come closest to exclusive responding (Row 15, Table 1), we found in place of the original three-to-one preference for Component 1 a preference on the order of five-to-one—still far removed from exclusive responding.

Other evidence does support the matching model's prediction of exclusive preference. In two investigations (Herrnstein, 1958; Herrnstein and Loveland, 1975) pigeons chose between keys that required different numbers of instrumental pecks for equal access to grain. Herrnstein (1958) indicated that the pigeons showed exclusive preference for the smaller fixed-ratio requirement when the other was three or four times as large. Few procedural details were reported, and it is conceivable that his results were affected by key-position preferences, especially in view of the fact that the pigeons still selected only one key when the instrumental requirement was the same on both keys. Herrnstein and Loveland (1975) found exclusive preference for the smaller variable-ratio requirement when the other was twice as large or more. They provided partial control for position bias by arranging that the component with the smaller requirement appear on the left-hand key in all tests of one concurrent schedule, and on the right-hand key in all tests of the next schedule. A possible problem with this control procedure is that the measure of preference associated with any particular schedule represents the combined influence of component and positional preferences. But exclusive preference has also been reported with a control procedure similar to

ours: Striefel (1972) found that humans showed an exclusive preference for the smaller of two instrumental requirements that differed by a factor of two and that granted equal payoffs.

To the familiar phyletic and procedural variables that might account for the difference between our results and those more favorable to the matching model (Bitterman, 1965), it may be necessary to add a new one. Recent evidence suggests that the discriminability of two fixed-ratio components depends in part on the absolute size of their instrumental requirements (Hobson, 1975). Such evidence raises the illustrative possibility that if the alternative requirements are 10 *versus* 40, the subject may be more likely to select the component with the smaller requirement than if the alternatives are 2 *versus* 8. The pattern of evidence already supports this speculation, because a common factor in studies that have reported exclusive preference is the use of instrumental requirements larger than those used in the present experiment. For example, Herrnstein and Loveland (1975), Striefel (1972), and we used schedules in which one component required four times as much instrumental responding as the other. Both requirements were relatively small in the present experiment, $I_2/I_1 = 8/2$. Both were relatively large in the experiment by Herrnstein and Loveland (1975)—48/12 or 96/24—and in the experiment by Striefel (1972): 40/10, 60/15, or 80/20. These variations in the absolute sizes of the requirements are not reflected by the ratio I_2/I_1 , which had a uniform value of 4, but are reflected by the sum $I_1 + I_2$, which varied from 10 in the present experiment to 60, 120, 50, 75, and 100 in the other two experiments.

It should be noted that neither the relative distance model favored by our results nor the matching model (Pear, 1975, Equation 9) attends to the absolute sizes of the requirements. Further experimentation may make it clear that a fully adequate model for selection must take absolute size into account, as well as relative size. Additional support for this position may be seen in an experiment by Hawkes and Shimp (1974), who found that pigeons responding for grain on variable-interval schedules displayed an increasing preference for the shorter of two behavioral patterns as the absolute duration of each pattern increased.

REFERENCES

- Allison, J. Microbehavioral features of nutritive and nonnutritive drinking in rats. *Journal of Comparative and Physiological Psychology*, 1971, **76**, 408-417.
- Allison, J. Contrast, induction, facilitation, suppression, and conservation. *Journal of the Experimental Analysis of Behavior*, 1976, **25**, 185-198.
- Allison, J. and Timberlake, W. Instrumental and contingent saccharin-licking in rats: response deprivation and reinforcement. *Bulletin of the Psychonomic Society*, 1973, **2**, 141-143.
- Allison, J. and Timberlake, W. Instrumental and contingent saccharin licking in rats: response deprivation and reinforcement. *Learning and Motivation*, 1974, **5**, 231-247.
- Bitterman, M. E. Phyletic differences in learning. *American Psychologist*, 1965, **20**, 396-410.
- Duncan, B. and Fantino, E. The psychological distance to reward. *Journal of the Experimental Analysis of Behavior*, 1972, **18**, 23-24.
- Ferguson, G. A. *Statistical analysis in psychology and education*. New York: McGraw-Hill, 1959.
- Hawkes, L. and Shimp, C. P. Choice between response rates. *Journal of the Experimental Analysis of Behavior*, 1974, **21**, 109-115.
- Herrnstein, R. J. Some factors influencing behavior in a two-response situation. *Transactions of the New York Academy of Sciences*, 1958, **21**, 35-45.
- Herrnstein, R. J. and Loveland, D. H. Maximizing and matching on concurrent ratio schedules. *Journal of the Experimental Analysis of Behavior*, 1975, **24**, 107-116.
- Hobson, S. L. Discriminability of fixed-ratio schedules for pigeons: effects of absolute ratio size. *Journal of the Experimental Analysis of Behavior*, 1975, **23**, 25-35.
- Morrison, S. D. The constancy of the energy expended by rats on spontaneous activity, and the distribution of activity between feeding and nonfeeding. *Journal of Physiology*, 1968, **197**, 305-323.
- Pear, J. J. Implications of the matching law for ratio responding. *Journal of the Experimental Analysis of Behavior*, 1975, **23**, 139-140.
- Schwartz, B. Effects of reinforcement magnitude on pigeons' preference for different fixed-ratio schedules of reinforcement. *Journal of the Experimental Analysis of Behavior*, 1969, **12**, 253-259.
- Striefel, S. Timeout and concurrent fixed-ratio schedules with human subjects. *Journal of the Experimental Analysis of Behavior*, 1972, **17**, 213-219.
- Timberlake, W. and Allison, J. Response deprivation: an empirical approach to instrumental performance. *Psychological Review*, 1974, **81**, 146-164.

Received 31 March 1977.

(Final acceptance 19 September 1977.)